

Student Critical thinking in Solving Two Dimensional Armetics Problems Based on 21th Century Skills

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Abstract— *Critical thinking has a very important role in constructing the improvement of students' ability to face the 21st century, especially in generalizing the pattern of the two dimensional arithmetic series. Since critical thinking is necessary when we try to understand and process information, put forward ideas or ideas objectively, and develop deeper insights. The purpose of this study is to analyse students' critical thinking skills based on P21. This study is a combination method in which this method is a combination of quantitative and qualitative methods. Research subjects are high school students. This subject is expected to provide an overview of critical thinking based on P21. Data collection techniques in this study are: (1) test, this technique is used to measure the ability of students in mastering the arithmetic array of two dimensions; (2) interview; it is based on the students work in solving the problem of two dimensional arithmetic series. The data result showed that in the experimental class had increased 40.85% on the indicator of effective reasoning, 37,44% on indicator of thinking system, 47,53% on decision indicator, and 42,55% on problem solving indicator. While the control class experienced a 19.5% increase in the effective reasoning indicator, 0.07% on the indicator of the thinking system, 0.02% on the decision making indicator, and 0.02% on the problem solving indicator.*

Keywords—*Critical Thinking on 21st Century Skill, Problem Solving, Problem-based Learning.*

I. INTRODUCTION

Critical thinking is an important aspect in mathematics is the ability to think critically. Critical thinking has a very important role in constructing the improvement of students' ability to face the globalization era. This is in line with 21st century learning (P21), which is learning that requires learners to have competence in critical thinking, creative thinking, communicative, and collaborative [1].

According to the book P21, critical thinking consists of (1) reasoning effectively, (2) thinking systems, (3) making decisions, and (4) problem-solving skills [1]. Zetriuslita (2016) defines to argue from facts that see the

lack of an argument, evaluate evidence and determine cause and effect [2]. The critical thinking that is intended in this study includes the activities of analyzing, evaluating, and providing solutions in every problem. Mason (in Lunenburg, 2011) states that the concept of critical thinking is one of the most significant trends in education and has a dynamic relationship in the learning process. Lunenburg added after understanding the thought content that produces, organizes, analyses, synthesis, evaluates, and transforms it [3]. Further, Facione (2011) argue that the basic concept of critical thinking is interpretation, analysis, evaluation, concluding, explanation and confidence [4]. Therefore, critical thinking is needed when we try to understand and process information, put forward ideas or ideas objectively, and develops deeper insights. Thus, critical mathematical thinking skills have an important role for students in constructing student abilities and finding the best alternative in solving problems.

According to Sharif, 2017, students need critical thinking skills when they face challenges by considering information received, making plans, deciding the right decisions, making decisions, and evaluating. Although critical thinking is very important, in fact, Indonesian students have not been able to develop their skills well [5]. This is evidenced by the achievement of Indonesian students' mathematics score in the PESA system is still relatively low compared to the average value of OEDC (Organization for Economic Co-operation and Development) in 2015 (Kemendikbud, 2016) [6]. Therefore, the researcher identifies that students' critical thinking skills need to be developed especially in solving arithmetic series problems since they affect the purpose and achievement of education in Indonesia.

Based on Fajarwati & Manoy (2017), giving a problem is one of the efforts that can be done to improve students' critical thinking ability of mathematical, since they will try to think to solve the problem by looking for the problem solution. One of the learning models that can improve the critical thinking skills of mathematics in learning mathematics is a model of problem-based learning [7].

As mentioned previously, the need for certain research is on critical thinking to face the 21st century. Thus, the researchers apply problem-based Learning to improve the students' abilities. The research results conducted by Oktavia Filda (2017), the goal of problem-based learning are to train students to be more independent in order to develop the ability to solve problems [6]. Nur Izzati Abdullah, Rohani Ahmad Tarmizi, Rosini Abu (2010) elaborate that the purpose of problem based learning focused on collaboration, communication, and problem solving skills [8]. While the objective of the research is to analyse the students' ability in solving the problem of the two dimensional arithmetic series based on P21st Century Skill through Problem Based Learning, in which the students will be able to reason effectively, use the thinking system well, make the right decision / argumentation, and arrange strategy / generalize in solving the problem.

Michael Angelo B. Promentilla, Rochelle Irene G. Lucas, Kathleen B. Aviso, Raymond R. Tan (2017) state that PBL is one way of developing students' learning skills in solving problems [9]. In line with Hobri's perspective (2009), students can develop thinking processes that include inductive, deductive, semiotic, analysing, criticizing, and making concurrent decisions and conclusions [10]. In the process of developing thinking skills through problem-based learning models, students are not only given a general knowledge, but rather bring them to a higher level of critical thinking and solve problems. The attainment of this level in learning theory is known as Higher Order Thinking Skills (HOTS) (Dafik, 2015) [11].

Many researches investigate in the mathematics education area, as well as arithmetic series. A two dimensional arithmetic array can be used to measure students' critical thinking skills since they require high-level mathematical skills especially in generalizing partition patterns. The two dimensional arithmetic array is the development of the arithmetic $Un = a + (n - 1)d$ into the following table, i denotes the position of the term in the horizontal row, whereas j denotes the position of the term in the vertical row.

$i \backslash j$	1	2	3	...
1				
2				
...				

The two dimensional arithmetic series (i, j) is on partitioning techniques. The symbol used is: $(P_-(m, d) \wedge n)$ where P is the partition, n is the number of columns, m is the number of rows, and d is the difference among the column rows called $P_-(m, d) \wedge n(i, j)$, which means

table / partition (i, j) with many n and m particular, and has a difference d in its sequence [12].

II. METHOD

This study is mix-method. Mix-method is a combination of quantitative and qualitative methods. Quantitative method is the collection of data whose data are numerical that can be quantified. Qualitative methods are used to understand and explore in-depth and process empathy, procedural, assessment, and evaluation activities. This triangulation approach is selected to provide an idea of the critical thinking skills level of students based on 21st century learning (P21) [13].

The subjects of this study were experimental class and control class in class of XI SMA consisting of 17 students. The subject of this study is expected to provide an overview of the ability to think critically based on P21. Then, the data is analysed, presented, and verified using triangulation approach in order to obtain valid data.

Data collection techniques used in this study were: (1) test, this technique is used to measure the students ability in mastering the arithmetic array of two dimensions; (2) interview, it is based on the students work in solving the problem of two dimensional arithmetic series. This technique is used to find out the reasons for the steps students used in problem-solving on the test.

The data presentation in this research covered classification activities and identification data to draw conclusions. In this study, the data exposure is the classification and identification of students' critical thinking skills. The indicator of critical thinking ability (CTA) that is reason effectively, use systems thinking, make judgments and decisions, and solve problems. The characteristic level of critical thinking is as follows:

Table.1: The Level of Critical Thinking Skill

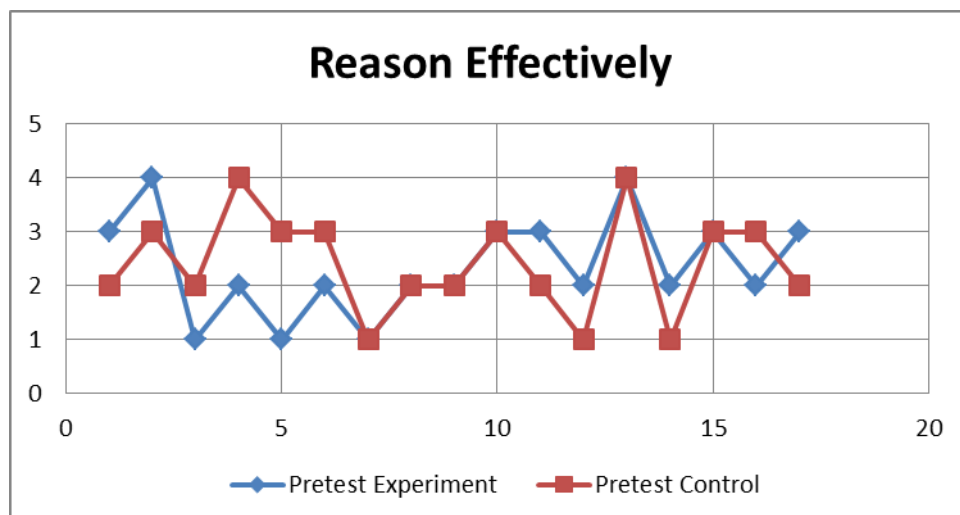
Level	Characteristics
Level 4 (Very Critic)	Students are able to clarify, evaluate the results of completion and interpret the result, and arrange the strategies to generalize each problem.
Tingkat 3 (Critical)	Students are able to clarify, evaluate the results of completion and interpret, but they have not been able to formulate strategies to generalize every problem.
Tingkat 2 (quite critical)	Students are able to clarify, but they have not been able to evaluate the results of

	completion and interpretation, and develop strategies to generalize each problem.
Tingkat 1 (less critical)	Students are only able to clarify problems but they have not been able to evaluate and interpret it, and formulate strategies to generalize every problem.
Tingkat 0 (not critical)	Students are unable to show all aspects of critical thinking in solving problems

III. FINDINGS

The first step in this study is to provide student worksheets and interview instruments which are validated by 2 validators, to be precise the first validator is Undergraduate lecturer and the second validator is Graduate lecturer. The researchers border the subject into a two dimensional arithmetic array (i, j) with partitioning techniques. In the second dimension table, i depend on the number of n , whereas j depends on the number of m that has been determined. Next, the students try to fill the table with numbers to find out the pattern of the series. Then, it is summing the numbers in each column that the sum of the numbers in each column must form an arithmetic

a) Indicator of reason effectively



According to the previous diagram, it can be seen that there are 2 students from the experimental class and 2 students are in very critical category, 5 students of experimental class and 6 students of control class are critical, 7

sequence. $P_{-}(m, d)^n$ used in this study includes $\{P_{-}(m, m)^n, P_{-}(m, m^2)^n, \text{ and } P_{-}(m, m/2)^n\}$ [12].

3.1 Determining the two dimensional arithmetic patterns

The first stage in this process, students are given the task to complete the table contained in the student worksheet and make the pattern n and m differently.

3.2 Summing up some two dimensional arithmetic patterns

The second stage, students are given the summing task of some patterns that have been presented precisely the first pattern and the second pattern, the second pattern and the third pattern, and so on.

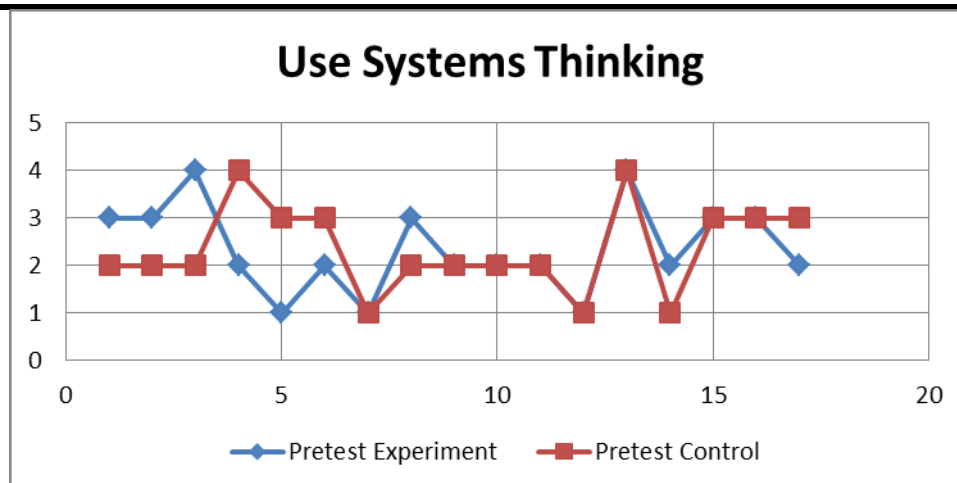
3.3 The analysis of research results based on pre-test and post-test

3.3.1 The experimental pre-test and control class results

The student result of pre-test problem solving, then, analysed to know the student level of critical thinking and problem solving based on P21 indicator on experiment class and control class so that it can be used as the basis of this study. The indicators are reason effectively, use systems thinking, make judgments and decisions, and solve problems. This following is the pre-test results based on the four indicators in question:

b) Indicator on systems thinking

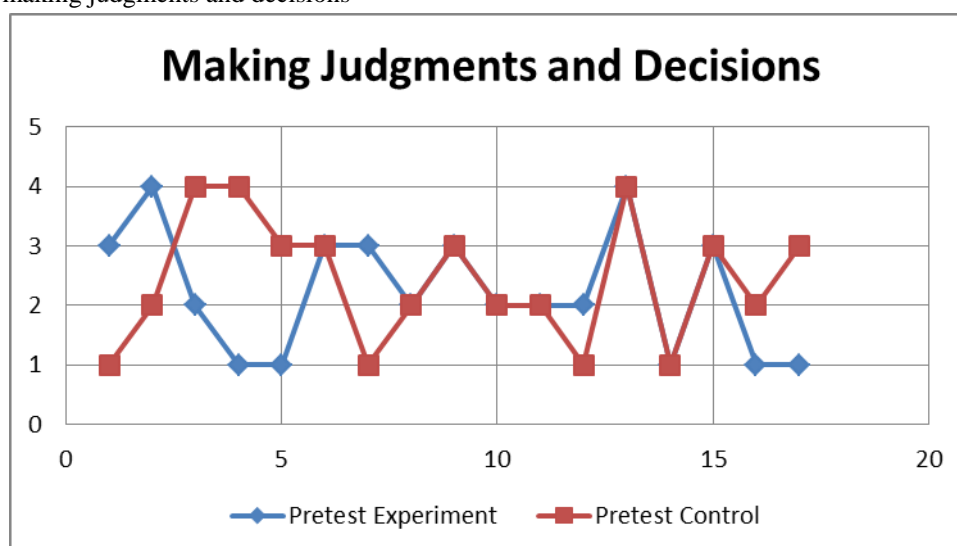
students of experiment class and 6 students of control class are quite critical, and 3 students of experiment class and 3 control class students are less critical.



The previous diagram can be seen that there are 2 students from the experimental class and 2 students are in category of very critical, 5 students of experimental class and 5 students of control class are critical, 7 students of

experiment class and 7 students of control class are critical, and 3 students of experiment class and 3 control class students are on less critical categories.

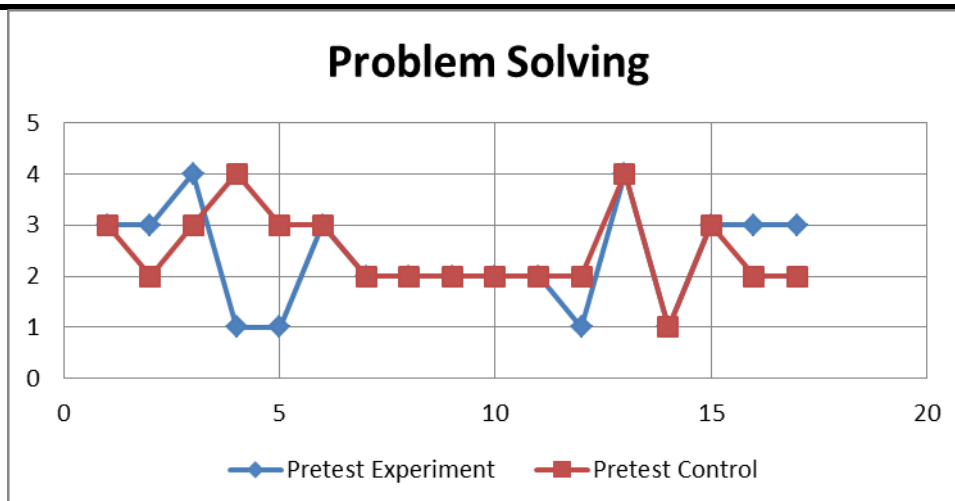
c) Indicator on making judgments and decisions



The previous diagram can be seen that there are 2 students from the experimental class and 3 students are in very critical category, 5 students of experimental class and 5 students of control class are in category of critical, 5

students of experiment class and 5 students of control class are critical, and 5 students of experiment class and 4 control class students are in less critical categories.

d) Indicator on problem-solving



It can be seen in the previous diagram that there are 2 students from the experimental class and they are in very critical category, 6 students of the experimental class and 5 students of control class are critical, 5 students of experimental class and 9 students of control class are

critical, and 4 students of experiment class and 1 student of control class are in less critical categories.

Based on the results, it can be concluded that among the experimental class and the control class has no significant difference. It was also proved by different test of both pre-test results on the following paired samples test:

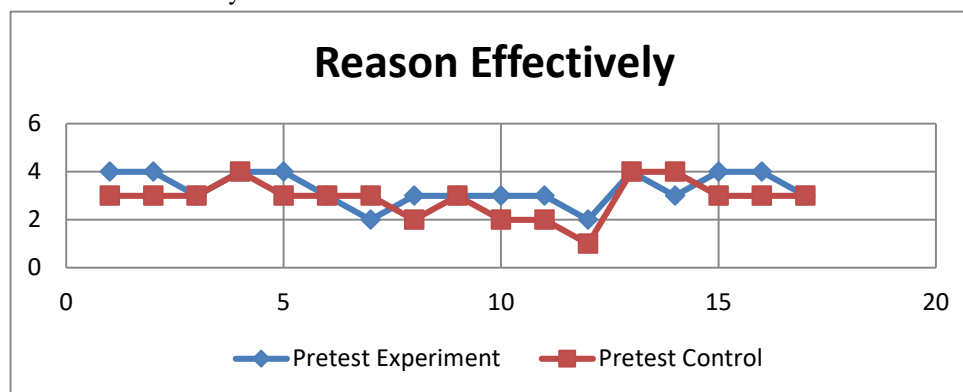
		Paired Differences							
					95% Confidence Interval of the Difference				Sig. (2-tailed)
		Mean	Std. Deviation	Std. Error Mean	Lower	Upper	t	df	
Pair 1	Exp_1								
	- Ctrl_1	-,11765	,99262	,24075	-,62801	,39271	-,489	16	,632
Pair 2	Exp_2								
	- Ctrl_2	,00000	,86603	,21004	-,44527	,44527	,000	16	1,000
Pair 3	Exp_3								
	- Ctrl_3	-,17647	1,46779	,35599	-,93114	,57820	-,496	16	,627
Pair 4	Exp_4								
	- Ctrl_4	,00000	,61237	,14852	-,31485	,31485	,000	16	1,000

From the different of test results, it can be seen that each indicator of the pretest has Sig (2-tailed) > 0, 05. It is concluded that there is no significant difference between the experimental class and the control class [14].

3.3.2. Post-test result of experiment class and control class

The result of the students' post-test on the experimental class and the control class in solving the problem was analysed to find out the critical thinking ability based on P21 indicators referring to some indicators that have been formulated. This following is the post-test result based on the indicator in question:

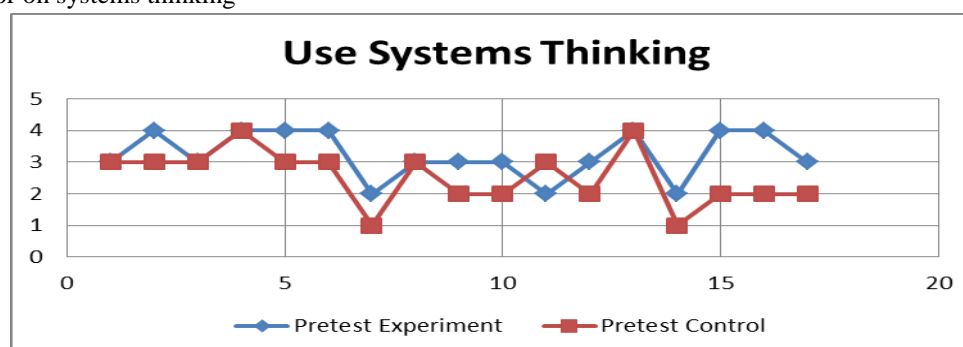
a. Indicator of reason effectively



According to the previous diagram, it can be seen that there are 7 students from the experimental class and 3 students are in the category of very critical, 8 students of experimental class and 10 students of control class are

critical, 2 students of experiment class and 3 students of control class is critical, while only 1 student in control class are less critical categories.

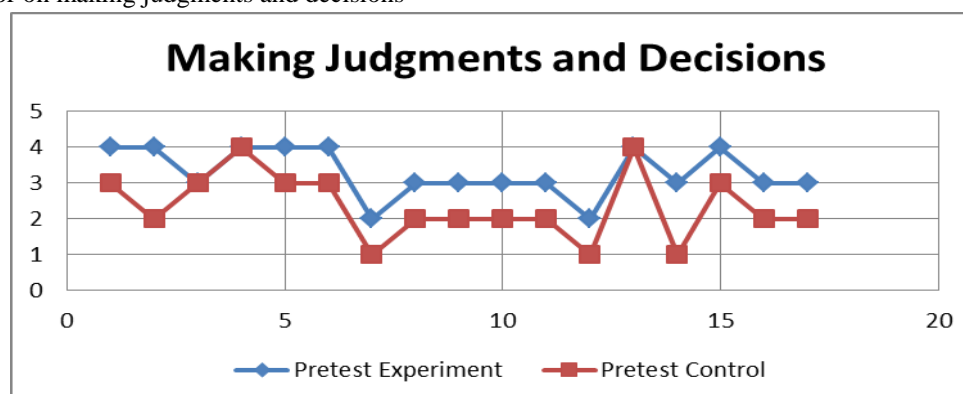
b) Indicator on systems thinking



The diagram presents that there are 6 students of the experimental class and 2 students are in very critical category, 7 students of experimental class and 7 students of control class are critical, 3 students of experiment class

and 6 students of control class is quite critical, while 2 students only in control class are in the category of less critical.

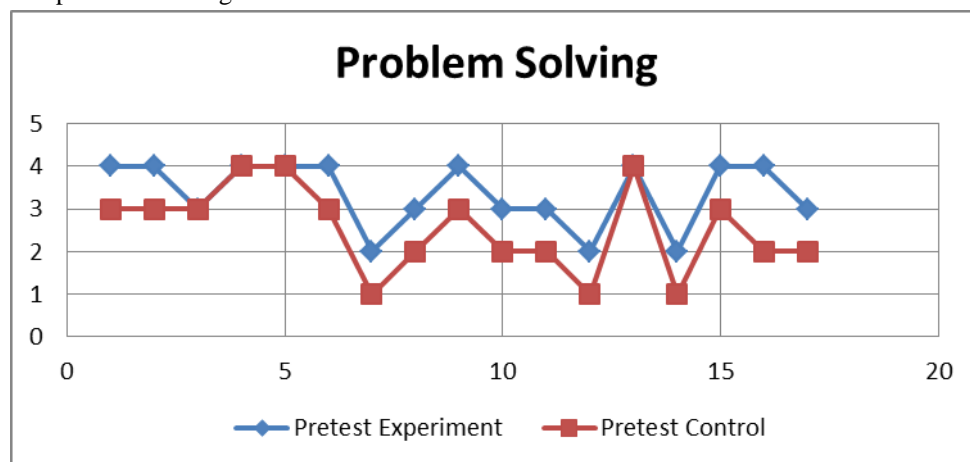
c) Indicator on making judgments and decisions



The diagram shows that there are 7 students of the experimental class and 2 students are in very critical category, 8 students of the experimental class and 5 students of control class are critical, 2 students of the

experimental class and 6 students of the control class are quite critical, while the 3 students are only in the control class are in the category of less critical.

d) Indicator on problem-solving



The diagram indicates that there are 9 students of the experimental class and 3 students are in very critical category, 5 students of experiment class and 6 students of control class are critical, 3 students of experiment class and 5 students of control class is quite critical, while 3 students only in control class are in the category of less critical.

Based on the results, it can be concluded that the experimental class has significant differences. However, the control class does not have the difference among pre-test and post-test. It is also proven by the test on paired samples test as follows:

Paired Samples Test

		Paired Differences							
		Mean	Std. Deviation	Std. Error Mean	95% Confidence Interval of the Difference		t	df	Sig. (2 tailed)
					Lower	Upper			
Pair 1	Exp_1 - Exp_post1	-,88235	,99262	,24075	-1,39271	-,37199	-3,665	16	,002
Pair 2	Ctrl_1 - Ctrl_post1	-,47059	,94324	,22877	-,95556	,01438	-2,057	16	,056
Pair 3	Exp_2 - Exp_post2	-,76471	,90342	,21911	-1,22920	-,30021	-3,490	16	,003
Pair 4	Ctrl_2 - Ctrl_post2	-,11765	,69663	,16896	-,47582	,24053	-,696	16	,496
Pair 5	Exp_3 - Exp_post3	1,00000	1,22474	,29704	-1,62971	-,37029	-3,367	16	,004
Pair 6	Ctrl_3 - Ctrl_post3	,05882	,65865	,15975	-1,39271	,39747	,368	16	,718
Pair 7	Exp_4 - Exp_post4	-,88235	,99262	,24075	-,95556	-,37199	-3,665	16	,002
Pair 8	Ctrl_4 - Ctrl_post4	-,23529	1,09141	,26471	-1,22920	,32586	-,889	16	,387

IV. DISCUSSION

The followings are student results in problem solving on summing several series of two dimensional arithmetics.

1. Subject on Experiment Class, Irfan Odiawan

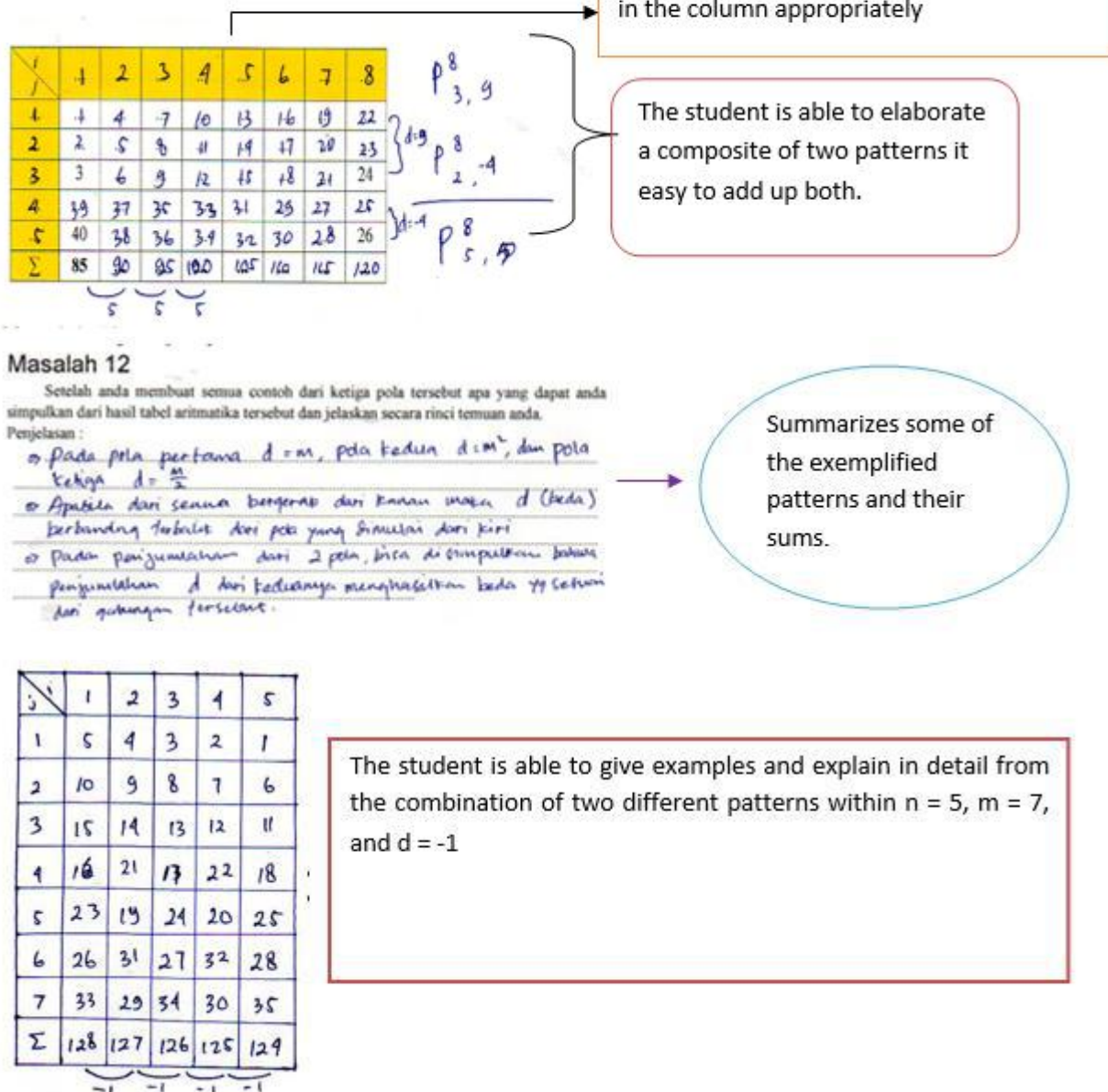


Fig.1: The result of Problem Solving in Experiment Class

Meanwhile, the student interview results on the experimental class during the process of summing two arithmetic patterns that are categorized are able to achieve the overall critical thinking indicator:

Teacher : Did you face the difficulties on summing the problems?

Student 1 : No

Teacher : What pattern that you take on summing the problems?

Student 1 : The second pattern, m 1-3 and m 4-5 patterns on the second one but it starts from the right.

Teacher : From the combination of the two patterns, which partition do you add up?

Student 1 : I only add up m and n .

Teacher : Why so?

Student 1 : because the summation rule of arithmetic is down.

Teacher : what can you conclude from the sum of the partitions?

Student 1 : before adding the two partitions, first, I must determine the pattern and know the value d of the two patterns. From the summation, the summation result of m and d are both corresponding that m and d are combined.

Teacher : can you create a new summation pattern with $d = -1$? Then explain!

Student 1 : for m 1 to 3 I take the first pattern that moves from right to left so that $d = -3$ and m 4 to 7, I take

the third pattern so $d = 2$. If both add up the result to $d = -1$.

Based on Figure 1 and the interview results, it can be concluded that the problem solving from one of the students in the experimental class is very critical. It is evident from the way students fill columns, summarize, and create a different partition sum with the previous

example. According to Ary Woro, the students having critical skills are students who are active in analysing their thinking, forming systematic planning in problem solving, using intuitive high in searching for information [15].

This followings are the student results in problem solving in summing several series of two dimensional arithmetic.

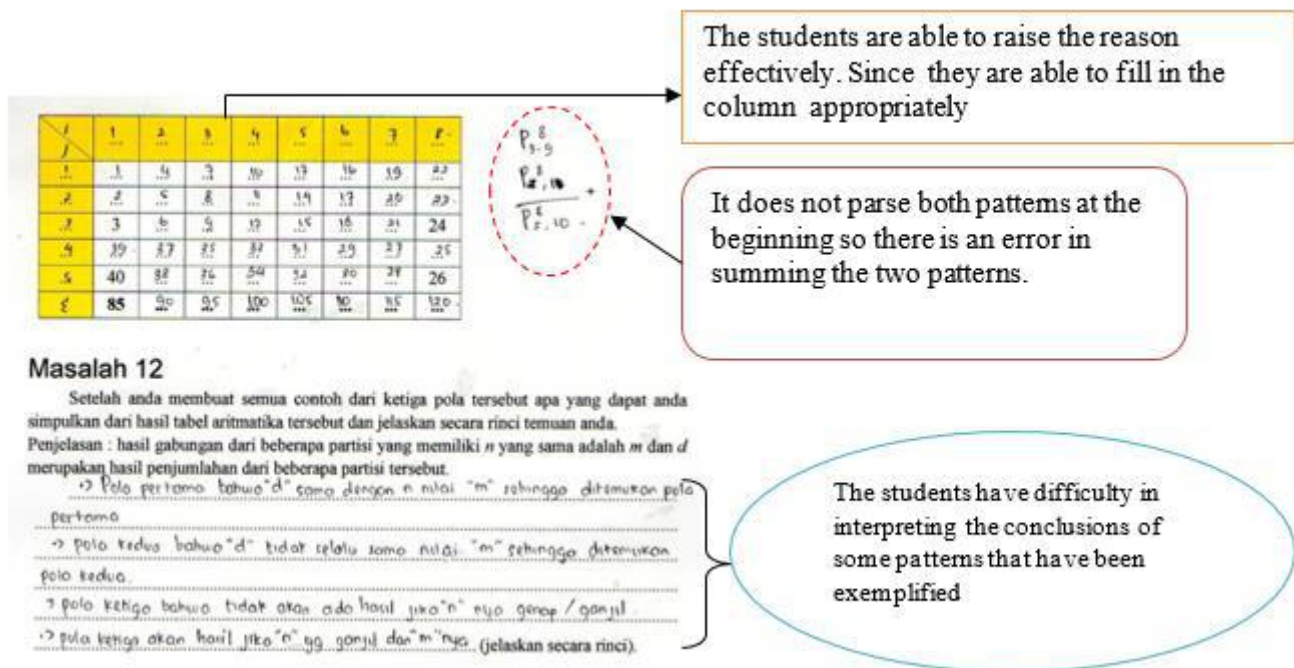


Fig.2: The result of Problem Solving on Control Class

Meanwhile, the interviews results for the students in the control class during the process of summing two arithmetical array patterns categorized as capable of meeting the overall critical thinking indicator:

Teacher : Did you face difficulties in summing the two patterns?

Student 2 : No

Teacher : Which pattern did you take for solving the problem?

Student 2 : The second pattern

Teacher : According to the two patterns, which partition did you sum up?

Student 2 : I only sum up m and d

Teacher : Why?

Student 2 : Because it cannot sum up.

Teacher : What can you conclude from the partition?

Student 2 : d in the second summation is different from d in table.

Teacher : Can you create a new sum pattern with $d = -1$? Then explain it!

Student 2 : Not yet, because I am still confused to determine from some patterns

Based on Figure 1 and the interview results, it can be concluded that the problem solving from one of the students in the experimental class is less critical. It is proved by the way students solve problems. Since students in this category have developed thinking skills. However, the ability to think is still limited to dig information that meets the standards of intellectual reasoning, digging and developing awareness of concepts and ideas that meet the standards clearly.

V. CONCLUSIONS

Based on the results, it can be concluded that the experimental class has increased as much as 40.85% on the indicator of effective reasoning, 37.44% on indicators using thinking systems, 47.53% on decision-making indicators, and 42.55% solve the problem indicator. While the control class experienced 19.5% increase in the effective reasoning indicator, 0.07% on the indicator of using the thinking system, 0.02% on the decision making indicator, and 0.02% on the problem solving indicator. Mathematically, the experimental class is higher in achievement than the control class.

Appendix: Tasks

Completing the Partition Pattern

Observe the following arithmetic pattern and find the first pattern through the pattern examples.

$i \backslash j$	1	2	3	4
1	1
2	...	6	...	8
3	9	...	11	...
Σ	15

$n = 4$ dan $m = 3$

$$p_{m,d}^n = p_{3,3}^{\dots}$$

$i \backslash j$	1	2	3
1
2	6
3
4
Σ

$n = 3$ dan $m = 4$

$$p_{m,d}^n = p_{\dots, \dots}^3$$

$n = 6$ dan $m = 5$

Observe the arithmetic pattern below and find the second pattern through the pattern examples

$i \backslash j$	1	2	3	4
1	1
2	...	6	...	8
3	9	...	11	...
Σ	15

Observe the following arithmetic patterns and find the second pattern through the pattern examples:

$i \backslash j$	1	2	3
1	1	4	2
2	5	3	6
Σ

$i \backslash j$	1	2	3	4	5
1	1	...	2
2	5	...
3
4	20
Σ

$n = 5$ dan $m = 6$

$$p_{m,d}^n = p_{\dots, \dots}$$

Determining the Partition

Look at the following table:

$i \backslash j$	1	2	3	4	5
1	1	11	2	12	3
2	13	4	14	5	15
3	6	16	7	17	8
4	18	9	19	10	20
Σ	38	40	42	44	46

$i \backslash j$	1	2	3	4
1	1	2	3	4
2	8	7	6	5
3	9	10	11	12
4	16	15	14	13
Σ	34	34	34	34

$i \backslash j$	1	2	3
1	1	5	7
2	8	6	2
3	3	4	10
4	12	11	9
Σ	24	26	28

Are the three tables partitions? Explain it in detail!

Summing Up From Two Partition Patterns

$i \backslash j$	1	2	3
1	...	2	...
2	6
3	7
4	12
Σ	20	26	32

$$\begin{array}{c}
 P_{\dots\dots\dots} \\
 P_{\dots\dots\dots} + \\
 P_{\dots\dots\dots}
 \end{array}$$

$i \backslash j$	1	2	3
1	...	2	...
2	6
3	7
4
5	15
Σ

$$\begin{array}{c}
 P_{\dots\dots\dots} \\
 P_{\dots\dots\dots} + \\
 P_{\dots\dots\dots}
 \end{array}$$

After you have created all the examples of the three patterns what you can conclude from the results of the arithmetic table and explain in detail your findings.

Explanation:

Applying Patterns and Joint Partitions

To better understand the three types of patterns, the arithmetic sequence pattern starts from the right (last column), and the combination of patterns, let's try to solve the following problems:

1. Create some two-dimensional arithmetic tables that yield $d = 4!$
2. Apply the combined method of the patterns to the following problem. Give some combined examples of:
 - a) 2 patterns that yield $d = 2$
 - b) 3 patterns that yield $d = -1$

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